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ABSTRACT

This paper is addressed to those who are concerned with the development of the school curriculum as a whole and to those who have a particular interest in mathematics. The aim is to promote discussion between these two overlapping groups, so that the growing use of computers in mathematics can be coordinated with developments in other areas of the curriculum. The paper resulted from a series of visits to schools in England by one of Her Majesty's Inspectors. Topics discussed include the basis for the school curriculum; the place of mathematics in the curriculum; the Cockcroft report; current uses of computers in schools at both elementary and secondary levels; types of uses, including drill and practice, short and simple programs, and games; student reactions; background on computer studies; project ; computer familiarization; mathematics and other subjects; the relationship of mathematics with computers; and the implications for mathematics teaching and for resources, staffing, and in-service training. (MNS)

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Microcomputers and Mathematics in Schools

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FOREWORD

This paper is addressed to those who are concerned with the development of the school curriculum as a whole and to those who have a particular interest in mathematics. The aim is to promote discussion between these two overlapping groups, so that the growing use of computers in mathematics can be co-ordinated with developments in other areas of the curriculum. It results from a series of visits to schools in many parts of the country and from consultation with a number of HMI with a variety of interests.

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MICROCOMPUTERS AND MATHEMATICS IN SCHOOLS

Introduction

1. Many changes have taken place in the school curriculum in recent years and mathematics has been one of the areas most involved. Its situation has been reviewed and recommendations made in the Cockcroft report; but the rapid development in the educational use of microcomputers alters the circumstances once again. This paper describes what is currently to be seen in schools, and identifies issues concerning mathematics and its relationship with the rest of the curriculum which the growing use of microcomputers is raising, and about which decisions will have to be made in the very near future. It is the intention to assist discussion rather than to make recommendations.

2. The paper results in the first place from a series of specially arranged visits to schools in many parts of the country, made by an HMI working half-time over an academic year. The schools were chosen because there was an initial indication that interesting work was going on in them. In some cases joint visits were made with other HMI who worked in the region. However, no special effort was made to ensure systematic cover of schools known to be in the front line of innovation, such as development schools for national projects or schools where an LEA was mounting a special effort. Visits to these schools would, in many cases, have been premature, and we may expect their work to be appraised and disseminated when it is complete. Information was also gathered from visit notes covering the last two or three years, written by a variety of HMI, some specialists in mathematics or computer studies and some whose main interests were in other areas.

3. The changes which readily available personal computers make to the practice of mathematics are profound. The implications of current technology are so extensive that it is difficult to present a balanced appraisal without seeming to exaggerate. No doubt further developments are still to come, but whatever changes microcomputers may make to mathematical education in the future we may say, beyond all doubt, that some of the changes which they have made already are so big that we lack any previous standard by which to compare them. So far the changes in the

school curriculum are small, and the changes in mathematics syllabuses are very small indeed, but in some schools changes in the methods of teaching mathematics can already be seen and these could develop rapidly. The greatest change of all, however, is in the response of the pupils. Whilst this can be seen in mathematics lessons or in lessons timetabled as computer studies, even stronger evidence is to be seen in the activities which go on outside normal lesson time. Pupils in surprisingly large numbers are finding a joy and zest in some aspects of mathematics which they did not find before.

4. This reaction is not of course universal, and it is not to be found in the majority of schools, but the changes which have taken place so far affect at least as many schools as the changes of the 1960s did after a comparable interval of time, and that was a period of intensive curriculum reform. Furthermore, pupils are displaying this new-found enthusiasm in schools which are in no way unusual in respect of staffing, catchment area or material provision. It is possible to generate this splendid response in schools in which the teaching did not previously display any special character or distinction, but the response is by no means automatic and it does not arise merely because facilities become available. Some circumstances promote its growth much more than others.

5. This enthusiasm should be turned to good effect without delay. In order to do so it is necessary to relate these developments to the whole of the school curriculum, to the changes which are beginning to take place in the teaching of mathematics as a result of the Cockcroft report, and to the changes in the examination system at 16-plus which are currently under discussion.

What is the basis for the school curriculum?

6. The curriculum is structured very differently in primary and secondary schools. The most evident differences arise from the specialist teaching in secondary schools; because of this alone very different strategies are called for in primary and secondary schools whenever changes are planned.

7. As a result of an extensive survey of primary schools conducted from 1975 to 1977 HM Inspectors concluded that in schools as a whole the curriculum was probably wide enough to serve the needs then current. The demands of society (they said) were likely to rise, with literacy and numeracy remaining matters of great interest within which priorities may change. The immediate aim, especially for the average and more able pupils, should be to take what is done to greater depth rather than to introduce content that was new. It was important to make full use of teachers' strengths and to build on the existing knowledge of individual teachers without losing the advantages associated with the class teacher system. Some five years later this remains a reasonable standpoint from which to approach the problems of introducing computers into the primary school.

8. For many years there has been evidence of strain in the secondary curriculum. In the words of *Aspects of secondary education in England* (the report on the National Secondary Survey, HMSO 1979), "schools have been subject to increasing, sometimes conflicting, pressures and many of them have responded as best they can within a framework which itself calls for reconsideration". There are problems of complexity, balance and coherence, and when further modifications of the curriculum are discussed they must be considered in relation to the range of existing problems to which we are still seeking solutions.

9. It is no solution to the existing problems of curriculum planning to increase still more the range of available subjects. To quote *Aspects* once more, "the principle that some choice is desirable is scarcely open to question. Much more doubtful is the apparently widely accepted corollary that the more choice there is the better".

10. We may give a few examples of the problems arising from the present range of subjects by way of illustration; but this in no way exhausts the list. These questions all concern the age range 11–16. How many children should continue to study a foreign language for five years of secondary education; how many should embark on a second foreign language, when, and for how long? How is the study of physics, chemistry and biology to be reconciled with a balanced programme which gives proper time to arts and the humanities? What practical subjects should be fitted into the timetable of pupils who have a strong talent for academic studies, and

in what manner should these practical subjects be approached? What relationship should studies described as "craft, design and technology" have to the traditional crafts and sciences? Is there a case for introducing the study of economics or politics as examination options?

11. In addition to these long recognised, but still unresolved, questions there are now some more. Should pupils be encouraged to undertake separate examination courses in mathematics and computer science? With how many other sciences is such a combination compatible? Is the study of statistics desirable—if so where is it to be fitted in, and for how many pupils? What should be the relationship of mathematics, statistics and computer science to one another and with the subjects which use these disciplines as a means to an end?

12. These matters cannot be decided unilaterally; they need to be considered within some overall framework. At the moment no such agreed framework exists, but frameworks have been in debate for some time and some measure of agreement has been achieved. The HMI papers *Curriculum 11–16* (DES 1977) and the later publication *A view of the curriculum* (HMSO 1980) provide a well discussed basis for a consideration of the place of mathematics in the courses which schools provide.

13. The heart of the thesis argued in *Curriculum 11–16* is that the period of compulsory schooling is concerned with introducing pupils to certain 'areas of experience'. These areas are, in alphabetical order,

the aesthetic and creative
the ethical
the linguistic
the mathematical
the physical
the scientific
the social and political
the spiritual.

The intention is that this should serve as a checklist for curriculum analysis and construction.

14. A discussion in terms of areas of experience may seem more remote from the practice of schools than a discussion in terms of subjects, but planning the curriculum in terms of subjects which steadily increase in

number leads to the irresolvable tensions to which we have drawn attention. Thinking in terms of areas of experience provides a framework which is not continually expanding. This does not presuppose that the school timetable will necessarily be drawn up in terms of areas of experience, and it would be very unrealistic to suppose that 'subjects' will be replaced by something else. This list is a stimulus to strategic thinking which provides guidelines when undertaking the more tactical tasks of planning subject syllabuses.

The place of mathematics

15. The list above is not presented in order to argue that mathematics should have an assured place in the curriculum as 'area four'. The list challenges us to ensure that we include in the area of experience called 'mathematics' the range of elements that a properly balanced curriculum demands; it challenges us also to ensure that proper relationships are established with neighbouring areas. The need to have a good relationship between the teaching of science and mathematics has long been recognised, although experience shows that it is difficult to achieve. Mathematics also has linguistic, aesthetic and creative aspects, the study of statistics is of importance in the social and political area, and there are relationships with other areas as well.

16. What is the place of computer use in this scheme? If we take a subject view of the curriculum it is clear that computers may be used with benefit in many subjects, quite apart from any claim that 'computer studies' should exist as a subject in its own right. Within this complex pattern of inter relationships we need to discuss the role of the related disciplines of mathematics, computing and statistics. If a workable basis for the curriculum is to be established it may be necessary to forsake the subject model for a model based on areas of experience. If this is done we have to ensure that the mathematical area of experience is defined in a broad enough way for it to play a full part in providing a complete education. If the curriculum is to be planned around areas of experience then the area of mathematical experience must be more extensive than the area defined by current mathematical syllabuses; it must include some aspects of computer use and statistical analysis; and it must be planned not simply as a self-contained discipline but planned in relation to the other areas of experience as well.

The Cockcroft report

17. The Cockcroft report, *Mathematics counts* (HMSO 1982), devoted a separate chapter to calculators and computers. The committee considered that the availability of low cost calculators and microcomputers had very great implications for the teaching of mathematics. The two fundamental issues are how these machines can be used to assist and improve the teaching of mathematics, and the extent to which the availability of the machines should change the content of what is taught or the relative stress placed on the different parts.

18. Public concern that the use of calculators might inhibit the proper development of computational skills was discussed, but the committee was of the view that the weight of research evidence indicated strongly that the use of calculators has not produced any adverse effect on computational ability. The availability of a calculator in no way reduces the need for mathematical understanding on the part of the person using it. Good habits need to be taught, and answers should always be checked by suitable estimation and approximation. The fact that not all calculators work the same way is no reason for avoiding them in the classroom. On the contrary, the comparison of ways of performing the same calculation on different machines can be very instructive. The committee recommended that examination boards should design their syllabuses on the assumption that all candidates will have access to a calculator by 1985.

19. Although only four pages of the report are devoted to computers, this was certainly not intended as a measure of the relative importance of computers in mathematical education. It reflects, rather, the limited amount of information which was available to the committee when the report was submitted to ministers in November 1981. This is one more reminder of how rapid and how recent the increase in the availability of microcomputers has been.

The report discusses the position of computer studies in the curriculum, reminding us that this started in the 1960s and was developed very largely by teachers of mathematics. The view was unanimous among the experts in the field of computer studies who were consulted that computer studies should not be regarded as part of mathematics but should ideally exist

within a separate department. (The committee attributed this view to the experts who were consulted rather than stating this view as their own.) The Cockcroft report continued by pointing out that the number of specialist teachers of computer studies is very small indeed, and that it seems inevitable that for the time being, and probably for some time to come, a significant proportion of the teaching of computer studies will be undertaken by mathematics teachers, who will in consequence have less time available to teach mathematics.

20. When the report was written the time devoted to the teaching of computer studies was probably equivalent to the teaching commitment of at least 600 mathematics teachers. Now this number must be very much greater because the teaching of computer studies has grown considerably in the last two years and continues to do so.

21. The report makes it very clear that as far as manpower is concerned the staffing of mathematics and the staffing of computer studies are not independent issues. They need to be considered together; and this is true also when we allocate time in the overall curriculum.

22. In a short space the report packs much useful advice on using computers to enhance the quality of mathematics teaching in both primary and secondary schools. This advice is fully consonant with the views expressed elsewhere in this paper and it need not be summarised here.

What is currently to be seen in schools?

23. Considerable use is being made of microcomputers in secondary schools and there is a rapidly growing interest in primary schools which has already gone beyond the pioneering of early enthusiasts. The manner of use varies greatly; it is not easy to give a tidy classification, and even less easy to attempt to quantify the various types of usage which occur. Many of those who argue for the introduction of microcomputers into the classroom stress that these machines have a contribution to make in many areas of the curriculum, and there is already evidence for this to be seen in classroom practice. Some advocates of computers express a more extreme view and say that computers should not be associated particularly with mathematics, and even that such association should be resisted and links between the two diminished.

24. Most primary classrooms are organised in ways which facilitate individual work or work in small groups. In consequence there is a noteworthy difference between primary and secondary schools. Where only one machine is in the classroom, in the primary school it is more likely to be in the hands of children, whereas in the secondary school it is more likely to be in the hands of the teacher. Many secondary schools have classrooms containing a number of microcomputers and in this situation the machines are, of course, being used by pupils; either on a common set task or individually. Comparable situations are not to be found in primary schools, although there are children of primary age in middle schools, some of which already have a teaching room with ten or more machines.

Primary

25. Young juniors and even infants have no particular difficulty in operating microcomputers. They can get the machines started, load tapes and overcome minor problems without needing to consult the teacher. Children are good at helping one another and some clearly take pride in their adroitness with this new equipment. Work was seen with programs which teach English language, with instructional games of various kinds, and with programs cultivating the ability to classify. A high proportion of the activities seen were mathematical. These range from drill and practice on number bonds to programs presenting number activities which encourage algebraic generalisation. A number of programs are geometrical; these most commonly encourage an active approach to geometry with tasks such as manoeuvring a point round a maze, using the machine to produce complex drawings or to explore systematically generated patterns.

26. On the value of drill and practice and on the best strategies for them teachers' views vary. Most children do not acquire numerical skills without some measure of deliberate practice, nor do they maintain their skills without periodical revision. But skills need to be built up in relation to children's previous knowledge and they need to be applied in a variety of contexts. The learning of mathematics requires a certain amount of carefully planned drill and practice and there are difficulties at both extremes. Major difficulties associated with excessive drill and practice are boredom and the feeling of

inadequacy, anxiety or even panic associated with direct questions which the child is unable to answer. Unhappy memories of such experiences colour the attitudes towards mathematics of many adults. A general advantage of computer-managed drill and practice is that anxiety is largely absent. At its best computer-managed drill and practice enables the child to compete with himself, with tasks matching his level of achievement, with rapid feedback, and with a confidential record of his scores available to the teacher at the end of the day. But if practice is to be of value needs have to be diagnosed, and there is no point in indiscriminate practice which is unrelated to the child's level of development. In these matters the teacher has to be the judge; as yet no software has been seen assisting in this role to any significant extent.

27. Helpful drill and practice were seen in a number of schools, but there remain many questions if programs which teachers have written for their own use are to be disseminated more widely. If a teacher writes her own programs they relate to the needs of the children as she assesses them and they fit the approach used in the classroom. But good programming demands knowledge, skill and time, and it would be foolish for every teacher to develop extensive programs from scratch. There may be value in software packages which provide frameworks into which teachers can put their own examples. At the present state of the art developments on these lines are very feasible.

28. Some of the more useful practice programs which have been seen in use in schools have been produced in unusually favourable circumstances, as when a professional programmer has been employed to write programs which the teachers designed. In some cases these programs have by no means been as effective when seen in another school where the circumstances were different and where the reasons behind the original design were not appreciated.

29. The microcomputer is a good medium for launching small-scale investigations on number and shape. In one school third and fourth year junior children were seen investigating a series of related problems. The machine displayed a particular tile pattern and the children had to decide how many tiles were needed to fill in the gaps. Other patterns of a similar type were displayed and the children had to decide what the numbers were in the different cases. Eventually they had to determine the general rule.

As the program used the random number generator of the computer a different sequence of examples occurred each time the program was run. The task was of a suitable level of difficulty, the children found it fun, and it provided a gentle introduction to algebraic ideas.

30. In some cases primary children were seen using programs which were designed for rather older children, with unhappy results. The teacher was pressing the children to deliver the 'right' answer when it was the intention of the program writers to encourage a freer, investigational approach with the teacher and class deciding which lead to follow up. When the activity was treated merely as a guessing game and there was no discussion and no refinement of the guesses nothing was being learnt.

31. Perhaps the most ambitious mathematical activities to be seen in primary schools arise from the use of the programming language LOGO. LOGO was first developed in the USA some 20 years ago and it exists in a variety of forms. The developments to be seen in English schools at present appear to be restricted to the parts of the language concerned with 'turtle geometry'. This enables a cursor to be moved around the screen by various commands such as FORWARD, TURN. The amounts of turning and the lengths of step can be set by the user. As the cursor is moved line patterns can be drawn. These can be built up into sub-routines which can then be reproduced in their entirety by a single command to the computer. In this way children can draw elaborate designs very accurately. This resource can be used in various ways, and it has been seen in lower secondary classrooms as well as in junior schools. Teachers sometimes provide work cards with patterns to be imitated, and sometimes encourage free exploration and design. This work produces very keen discussion, and the movements of children's hands, as they indicate to their fellows what is needed, are most impressive. When discussing rotations and displacement they display a degree of involvement which is hard to develop at this age by other methods of teaching.

32. It is not clear however how this is to be developed. LOGO has been extended by its inventors to provide a full computer language, but fuller implementations have not so far been available on the machines which schools have. The general graphics facilities on the machines available to schools are very powerful, and they enable the user to work with relative

displacements (moving on from where you are, as with LOGO), as well as with displacements within a fixed coordinate system. The commands on current machines are very simple indeed, and whilst these graphics facilities might not offer quite the same opportunities as a full implementation of LOGO they could perhaps be used more extensively than they are.

33. Experience in the past has shown the difficulties of ensuring a smooth transition from primary to secondary school, especially when innovative syllabuses are involved. Liaison should be encouraged, but LOGO is a valuable activity for primary children even if the follow-up in the secondary school cannot be guaranteed.

34. It was not easy to find activities in which children interrogated a data base on a scale which only a computer would make possible. It may be necessary to wait a little longer until such data bases are more readily available. Enterprising work was seen in one school where census data was analysed using a data handling program written by a secondary teacher working for the same LEA. Various questions were investigated, the histories of various families were compiled and a considerable amount of writing and graphical work developed naturally.

35. One form of data handling package which is quite popular is exemplified by the program ANIMAL. This enables the user to develop a tree-structure which grows as more types of animal are inserted. With each extension the user has to provide a question which will discriminate between the new entry and some animal already included. There is interest in watching the uncanny way the machine builds the structure up correctly, and in formulating suitable discriminating questions. Some children do not find this at all easy because they cannot distinguish between a *question* enquiring about a difference and a *statement* declaring the difference.

36. The prospects for the successful use of microcomputers in primary schools were summed up by one head, who expressed a widely held view when he said "it all depends on the programs which are provided". In the primary school this seems to be just as true for mathematics as for other areas of the curriculum, although in the secondary school the situation is essentially different. Secondary mathematics teachers can make extensive use of microcomputers 'just as they come out of the box'. This is because the BASIC programming language with which today's microcomputers are

provided is rich enough to cover extensive areas of mathematics, and is easier to learn than many things which are universally established as part of the secondary curriculum.

37. This distinction between the use of microcomputers in primary and secondary schools is not of course hard and fast. Primary children have been seen writing elementary programs in BASIC and picking up BASIC by typing up programs which they have found in magazines. Such typing up involves more than simply copying, at this stage a little encouragement works wonders and many children quickly learn how to adapt programs to do fresh things, or to use resources which they find on one type of machine but not on another. The most advanced programming to be seen had been started in a lunch-hour club. The pupils had progressed rapidly and, in some cases with the help of parents or an elder brother, they were producing a quite ambitious program to teach an aspect of work they had done in class.

38. In the secondary school it is a feasible aim to relate this activity to the systematic development of those parts of BASIC which implement the mathematics syllabus which the pupils are learning. Many children in junior schools are clearly capable of, and ready for, similar work, and this does not depend at all on the provision of sophisticated programs; it depends on the teacher knowing how to do the appropriate thing with the machine 'in its raw state'. It may be a long time before we can rely on having a sufficient number of primary school teachers with the knowledge and the confidence to implement any general policy of developing programming ability and mathematical knowledge side by side in a systematic way.

Secondary

39. In secondary schools practice is also very varied. Schools differ very much in the strategies they are adopting and in the extent to which they pursue the various possible lines of development; and the same is true of the teachers within a single school. The observations which follow describe not only mathematics lessons but also lessons in other subjects in which mathematical ideas were involved. These lessons were usually in science or computer studies.

40. Programs come from a variety of sources. A lot of material is available in some LEA resource centres, and many lessons use material which schools have obtained through this channel. In many cases teachers have written their own programs. It is common to hear the advice that program writing is a matter for experts and that teachers should leave the matter to others; but many mathematics teachers have demonstrated their ability to write serviceable programs which meet their needs at least as well as programs written elsewhere. These programs would often need foolproofing and better documentation if they were to be transferred to other users, but for the moment they serve their purposes excellently. In some cases pupils have written programs which teachers are able to use.

41. Some good lessons on graphical interpretation were seen, using programs produced by a national project. One of these develops around a cartoon character taking a bath. A graph is drawn showing how the depth of the water varies with time. At first viewing the program may appear to present no more than an overelaborated joke—but there is much more to it than that. The graph and the action can be seen simultaneously, or they can be shown separately, so that as skill is acquired the pupils can be asked to deduce the actions which produced a graph which they see. They can then check up as the actions are displayed. The sequence of actions can be varied according to the suggestions of the class, and then it can be replayed exactly as they had it the time before. This makes it a very adaptable resource quite unlike anything which has been available before. Some very good discussion has arisen from it, and pupils have shown an ability in graphical interpretation surprising at their age, or at their previous levels of attainment. This teaching material has enabled teachers to use discussion as a teaching strategy in mathematics lessons when they lacked the confidence to do so hitherto.

42. In another school further material from the same national project formed the basis for a discussion lesson of a different kind. A small class was subdivided into two teams who played a competitive game in which success depended on finding the proper mathematical strategy. This approach has been adopted with success by a number of program writers.

43. Other good discussion lessons were seen given by teachers using their own programs, as in one girls' school where teachers used programs

on statistics and on transformation geometry. The random number generator in the computer enabled sets of statistical values to be produced for analysis which were different each time, and the tedious arithmetic was removed so that the class could concentrate on the principle involved. The geometry program in this school was one of a large number which the teacher had written, all with very polished presentation.

44. Teachers elsewhere are writing similar programs which show how geometrical figures are changed by reflection, rotation or by shearing. Some teachers have written demonstration programs in which the possibilities of animation are exploited. For example, a program seen in a sixth form lesson showed how the parabolic motion of a projectile arises from compounding a uniform horizontal velocity with accelerated vertical motion.

45. The use of the microcomputer in statistics lessons may be expected to increase rapidly in the future, but it is perhaps a little surprising that more use has not been seen already, especially in the sixth form where it is particularly important to handle large amounts of data. Over the last few years calculators (especially programmable calculators) have transformed the problem of data handling in school statistics but this increased power has been little utilised. Many teachers of statistics still aspire to teach no more than the older style of examination question which was designed to be manageable with minimal calculating equipment.

46. However, some new uses for the computer have been observed in related fields. In a physics class the teacher and pupils had earlier carried out a dice experiment to simulate radioactive decay. With dice the simulation was slow and so in the lesson observed the process was speeded up by using random numbers from the computer. The whole sequence of ideas not only clarified the meaning of "half-life" in radioactive decay, it was also a very workmanlike example of mathematical modelling.

47. Simulations usually depend on the use of random numbers, but the power of the computer to generate "random" numbers should not be taken for granted. Is it clear to all pupils that the computer is really simulating a die or a coin? One teacher started a lesson by asking "what properties do we expect random numbers to have?". This proved a fruitful question and he did not proceed until the pupils had formulated their thoughts in a sufficiently precise way.

48. Many recent writers have stressed that the computer is not "just a number cruncher" and that its function as an information processor is even more important. Pupils have been seen doing small scale information processing on files of their own during computer studies courses, but, as with primary schools, it has been hard to find established files of any size being used in a way which merits imitation. There has been little time for the development of these ideas and better practice may develop in future. However, an impressive demonstration was given in a science laboratory, when a pH meter took readings as a chemical reaction progressed and the graph was drawn by a microcomputer as the experiment took place.

49. Advice against regarding the computer as "only a number cruncher" appears misplaced, because during many visits to schools it was difficult to find examples of the computer's capacity for extensive heavy calculation being exploited at all. It was certainly difficult to find cases of the computer being used to assist in the study of straightforward arithmetical properties of number. By this we mean such things as the properties of prime numbers, of sequences such as the triangular numbers, squares and cubes, and properties of divisibility. These ideas are little studied in schools but they are encouraged by the publications of the subject associations and by the more scholarly textbooks. In this area pupils may get involved relatively quickly with ideas which fascinated the great mathematicians of the past.

50. One excellent lesson was seen in which the teacher started the class on the task of summing the divisors of a given number, so producing a new number. (The number itself was not classed as a divisor, but *one* was so classified.) As this process was repeated chains of numbers were produced. This led to many questions. It led also to a great deal of mental arithmetic which was gradually transferred to the computer by composing a suitable program. So the lesson proceeded; and if the pupils were as busy on the problem afterwards as the inspectors were it generated an investigation of record length.

51. This last lesson was one of the few lessons in which the computer was used to do arithmetic for its own sake. The majority of mathematics lessons seen using computers centred on the graphics capabilities of the machines and used arithmetical computation only incidentally.

52. The major inspiration found in the graphics is understandable because microcomputers enable elaborate mathematical diagrams to be constructed with comparatively little time and effort, and they allow colour and movement as well. Many lessons were seen using computer graphics, and although this facility has only been available for quite a short time teachers are already applying it in a variety of ways. Some lessons using graphics under the control of the teacher were described above. Here are some in which the emphasis was on the pupils using graphics themselves.

53. One school had arranged a workshop in three-dimensional geometry for three classes of first year pupils. Three teachers pooled resources and organised pupils in groups, most of which constructed three-dimensional geometrical shapes in a variety of ways. As one activity pupils took turns to use the program called BUILD, in which a cube can be manipulated in three directions on the screen. In this way a perspective drawing such as an architect might sketch can be built up. This program can be used in a variety of ways ranging from freely inventive construction to the attempt to sketch a solid construction of cubes which has been provided as a pattern.

54. Lessons were seen in which pupils were being taught to use the BASIC graphics commands on a machine. This does not require any previous teaching of programming; it is possible to embark on interesting geometrical work right away. Most of these lessons were with fourth year pupils, but in the future many schools may start this work earlier. Some of the groups were of comparatively low ability, and the pupils displayed an unusual degree of application. Such lessons were given very successfully with eight or nine machines with classes of up to twenty.

55. Most pupils, whatever their level of ability, seem more deeply committed if the tasks they are set in the early stages of this kind of work are relatively open. Even if the task appears quite simple to the teacher—simply to draw a square for example—there are ways of going wrong and the pupil may not yet have learnt how to erase a line and start again. But if the instructions allow greater freedom the pupil can change course and create something fresh into which the previous lines fit. Straightforward failure is avoided and just as much is learnt in the long run.

56. Another teacher employed a quite different strategy. He was taking his class through a sequence of graphical techniques which were eventually to be put together to make a PAKMAN game. Whilst many computer games may appear trivial to adult observers the skills needed to design such games are certainly not trivial. In the lesson concerned the pupils had to learn the coding system by which graphics characters are produced on the screen, and in this way they became familiar with an application of binary notation and practised it assiduously for a definite purpose.

Drill and practice

57. As in primary schools a number of lessons were seen in which computers were used for drill and practice. They have been seen in use to practise elementary arithmetical and algebraic skills of various kinds. These skills were usually practised divorced from context and not infrequently there were a few seconds of some shooting, bombing or hazard-evading game as a reward. Much doubt has been expressed in the educational press as to the value of activities of this kind; however, it is clear that this approach will be explored thoroughly and teachers will be able to come to conclusions about its value.

58. When practice is required the computer can be a useful way of providing it. These machines are still novelties and with almost all of the practice seen so far morale has been good. But practice needs to be related to the learner's stage of development and carefully designed for the purpose in hand, and in the long run we may expect practice which is unsuitably prescribed to be as boring in this medium as in most others.

59. Some current programs keep records as pupils use them and enable teachers to have an up-to-date list of their pupils' achievements. Some use of this has been seen in schools where the teachers were involved in the design of the programs, but this is one of those facilities which have not always transferred well to other schools.

60. Very little work has been seen using elaborately written programs to provide direct instruction. Not much material is yet available, and programs are not readily transferable between different kinds of machine. Modern microcomputers are vastly more flexible than the teaching machines of 20 years ago and they are much more likely to provide a satisfactory vehicle for programmed learning; but so far there is no evidence from the classroom that programs providing direct instruction in mathematics to individual students have made any impact, or that they are a promising way forward.

Short and simple programs

61. It was more surprising to find that comparatively little use was being made of short and simple programs, made up on the spot to illustrate points arising in the normal development of the mathematics course. Programs incorporate a particular kind of mathematical thinking; a program can provide an active way of approaching a mathematical idea when the traditional way is static. Greek geometry for the most part conformed to a Platonic ideal and presented immutable configurations of ideas; notions of movement, for example in the discussion of congruence, were deliberately excluded. By contrast the approach to geometry by means of transformations deliberately evokes ideas of movement as a pedagogical strategy. In a comparable way it is established practice in algebra to use equations to indicate stages in an argument. The Royal Society's guidelines on mathematical notation state that the equals sign is to be regarded as a fully inflected verb, but the understanding is that it is in the indicative mood. In a computer program the equals sign is normally in the imperative mood, and statements are used which would be contradictory in normal algebra. For example $x = x + 1$ is an indication in some implementations of BASIC to increase x by 1 (the word LET that is obligatory in some versions of the language is optional in others). Considerations of this kind are not simply matters of detail, they make possible an alternative approach to the teaching of algebra.

62. The matter has been put in a different way--you know that you have really grasped a mathematical process if you can program a machine to perform it. This implies a style of teaching which, at the moment, is hardly to be found.

Response

63. The response which many pupils show when they have the opportunity to do work which appeals to them on these machines is quite remarkable. During the National Secondary Survey of 1975–78 HMI made visits to 384 secondary schools, and the mathematical provision which these schools made and the response displayed by their pupils were assessed and recorded. The response which some pupils are currently displaying is completely outside the range of assessment operating then. It is now possible to find schools where numbers of pupils arrive long before normal starting time and where they occupy all the available microcomputers until school begins. This insatiable appetite is displayed again through the lunch hour and in every available break, and it continues until such time as the caretaker clears the school in the evening. A few pupils, and even teachers, become so obsessively involved that their other studies suffer; but this is a danger which teachers usually recognise and try to avoid.

64. Positive attitudes can go home from school. An HMI was returning home after a visit to London. At an intermediate station two 14 year olds in school blazers boarded the train. One produced a scruffy notebook and the other asked who the work was for. HMI noted the poor standard of written work and the tendency to talk about the teacher rather than about the content of the homework. But a lively discussion developed. The boy with the notebook was trying to program his ZX81 in machine code to enlarge and scroll a message. He wrote the commands down very fluently. His companion asked questions and gave advice. There was however some confusion over the different machines to which this particular machine code would apply. We are confident that such conversation is widespread.

65. There are far more boys than girls among the dedicated enthusiasts and some schools have taken special measures to try to redress the balance. These measures include having 'ladies only' times in the computer room, or a special club to encourage the girls. In spite of these activities it is rare to find a school where the girls match the boys in enthusiasm for computers or in attainment, and where schools are successful in achieving this the reasons for that success should be studied and made known more widely.

66. Whilst this level of enthusiasm is widespread it is by no means an inevitable consequence of having computers in a school and it is by no means certain that it can be maintained. At the moment personal computers appeal because of their novelty, and this will gradually fade. There is more to the appeal than novelty and it is most important to understand the way in which this appeal operates, because there is a real danger that the institutionalisation of computers in the wrong way may turn pupils off. At the moment there is appeal in the informality of the activities and in the fact that an eager pupil can outstrip not only his fellows but also his teachers. The syllabus is not yet full of items which have to be covered, and examined, come what may—there is very wide choice and in a number of schools staff and pupils are finding that they can choose the computer activities which interest them and follow a line of argument through as far as they feel inclined.

67. If this splendid activity becomes too highly organised and if it becomes unduly confined by processes of assessment the vigour and zest will be lost, and courses will develop in which fundamental objectives are replaced by derived objectives in the way described in a number of areas of the curriculum in the National Secondary Survey.

Games

68. A further activity to be seen in classrooms is the playing of computer games. Pupils are busy on computers for many hours of the day, and most of the game playing is permissible recreation outside lesson times. It is easy to say that game playing should not take place at all during lessons, but to say this is to ignore the immense energy which game playing unlocks. A great many pupils are excited by games, and a large number are excited by the genuinely creative work involved in developing games programs for themselves, or in developing programs which demonstrate some attractive feature of the computer. The problem for teachers is to move from the obvious excitement of game playing to activities which are more educational in the longer term.

69. To what extent is the zest, the fun and excitement which surround so much computer activity in schools merely the fun of the amusement arcade? Are children staying late after school merely to play Space Invaders? To some

extent the answer is 'yes', but it would be wrong merely to dismiss the activities because of this. One school visited had an interesting plan of work with a lower ability fifth form. They had planned the week's mathematics lessons to cover a range of work. Some time was spent on the systematic practise of skills in arithmetic, some was spent on applying the skills to important everyday applications and one lesson was given to computer use which usually included playing games. The staff were certain that the goodwill which resulted yielded an ample dividend. Pupil-teacher relations were so improved that much more work was achieved in the remaining lessons than would have been obtained by the staff insisting on hard grind all the week.

70. For some young people the playing of computer games may be no more than a mindless addiction, but there is much more to games than this. Of their own accord many pupils pass on to the editing of games, to their modification and improvement. It is a quite common sight to see an enthusiast typing in a program which he has found in a magazine, while a friend dictates the symbols line by line. Games programs often involve computing techniques well beyond an O-level computer studies course and many pupils are so determined to have a version of the game that they will type up symbols which, in isolation, are quite meaningless for them. Finally the program has to run, and even with experts very few programs run first time. This means that the errors have to be located and put right. This whole sequence of operations offers many opportunities to teachers. When errors are traced there is the opportunity to point out the idea on which a particular line of the program depends. In circumstances such as these a pupil can readily appreciate the value of knowing some theory; because if you know how a command works you are better able to spot errors involving it.

71. Games provide a strong initial motivation and they are a route to more academic things, but they can make disproportionate demands on time and resources and some schools have found that the time comes when games have to be severely controlled or even prohibited. These schools have done this to make their limited number of machines available for more serious use. Furthermore, it is a genuine disadvantage that game playing often involves the repeated use of a few particular keys by players who get excited. The consequent repeated thumping damages the machine. Machines are rapidly becoming available in the home and in the future most of the game playing

may take place there—but we should ensure that the educational possibilities are not lost to the school.

72. By starting with games many pupils have acquired confidence, a desire to go further, and an ability to argue in a friendly constructive fashion with the teacher and one another. Some pupils acquire this from orthodox mathematics, but in comparison only a few. One teacher remarked that these days it was normal for her mathematics lessons to begin with three or four hands waving—the questions would all be about computer operation or programming even though the lesson was to be ordinary mathematics. The microcomputer is an unparalleled source of mathematical conversation.

73. In many computer lessons and in much of the spare-time activity it has been especially pleasing to see the unusual combination of competition and collaboration between the pupils. After school the competition can begin outside the classroom door—it is seen as jostling for position to get a machine before the others do. Some pupils are quickly way ahead of their fellows, but there is a splendid camaraderie and the expert will leave his program for a precious few minutes to help out someone in difficulty. Pupils of differing ability are able to work together; a good A-level candidate and a new sixth former on a City and Guilds course worked together on a program to draw algebraic curves. The classroom relationships have a tone that indicates a group of willing volunteers. It is a challenge to schools to harness this energy and good will.

Computer studies

74. Schools have long experience of computing. In the 'sixties some schools had computers discarded by industry, and they maintained these for some time and carried out pioneer work. A number of enthusiasts constructed small-scale demonstration computers, and many schools developed courses which depended on the cooperation of LEA administration offices or nearby institutions of higher education to process the punched cards which pupils produced. This kind of batch processing still continues.

75. Eventually examination courses in computer studies were established and examinations have been available at CSE, O-level and A-level for some years. The examinations taken at 16 and to 18 differ in kind; the CSE

and O-level examinations are described as computer studies and the A-level examinations as computer science. Initially the computer studies courses were often developed for Mode 3 CSE examinations, and some Mode 3 O-level examinations preceded the current Mode 1 examinations. The schools involved had the responsibility of developing syllabuses suited to their own particular circumstances. The first syllabuses placed a high value on programming and allowed the schools to use any computer language which was available to them. These early courses also included logic gates, the internal workings of a computer, binary numbers and number manipulation. A section on the history of computing was included in most courses and this usually covered the period from early cave drawings to electronic computing. History and mathematics have now disappeared from many syllabuses and existing computer studies courses are quite restricted in scope, concerning themselves with applications of mainframe computers rather than microprocessors. Little reference is made to control technology and work in this area is descriptive rather than practically based.

76. Because they were developed in circumstances different from those now prevailing most present courses now require revision. They do not encourage practical application of the new technology. Courses often deal in a superficial way with social implications which require a greater sophistication than is exhibited by the majority of 16 year olds. The emphasis is mainly on theoretical applications of computers with particular emphasis on automatic data processing, so that it is possible for students to pass without programming or even coming into contact with a real machine. Current practice takes little advantage of the exciting world of microelectronics in which we are now living and it will be a serious loss if the new 16-plus examinations remove those elements which made the original syllabuses so stimulating to both teachers and pupils.

77. The expansion of examination courses has been rapid, and they may well expand further; but the further expansion of examination courses cannot be a means of introducing all pupils to computers. It would not be possible with present limitations of manpower to envisage staffing such courses, and examination courses are designed for only some 60 per cent of the ability range in the 16-plus age group. Furthermore the inclusion of computer studies as an examination subject in a curriculum which already contains mathematics and three sciences, and which is beginning to include various

courses in technology, presents problems of balance. This discussion paper is concerned most particularly with mathematics and the use it makes of microcomputers. We do not presuppose that mathematics is entitled to any particular privileges when the curriculum is designed, but we do point to problems of balance which are becoming increasingly critical and which need to be resolved.

78. It is possible that within a decade most of the material now called computer studies will be subsumed by other syllabuses (eg business studies, social studies, mathematics). Computer *science* is a different matter, being a deeper study of the technological aspects and applications of computing. If computer studies were to disappear as an examination subject at 16-plus its replacement by an examination in computer science for those students who wish to specialise in this field would be a fresh question.

Projects

79. A further development to consider is the extension of computer projects as part of examination assessment at 16-plus. At their best current computer studies courses make a decided contribution towards providing the missing four ingredients from the six kinds of mathematical activity advocated in the Cockcroft report*. These missing ingredients are to be found particularly in the projects which pupils do as a necessary part of the computer studies examination course.

80. The projects encourage individual work over an extended period of time, and this can lead to very profitable discussion between pupil and teacher, and between pupils. Such benefits are not inevitable consequences of project work, but well conducted work in this style is conducive to them. As computers become increasingly available projects which use computers could develop in other areas of the curriculum, and mathematics would clearly be one favourable area in which to start. We do not suggest that such project work should be compulsory, but there are precedents for optional project work contributing to a candidate's examination grading, and ways could be found to extend this practice.

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* Paragraph 243 of the Cockcroft report recommended that mathematics teaching at all levels should include exposition, discussion, practical work, consolidation and practice, problem solving, investigational work.

Computer familiarisation

81. Recent visits to schools have shown that many schools are planning courses of computer familiarisation. These are intended most often for the first or second years of secondary school, and sometimes for the third. It is early to say what these courses will contain, but we may expect them to involve the use of computers in so far as the machines are available. They may also contain wider work on information retrieval from traditional sources (reference books, libraries) and from more modern sources (PRESTEL, television, etc), and reference to the social implications of the technological changes which are currently taking place. Lessons seen so far indicate that discussion of the social implications of computers, desirable though it may be, is not easy to carry out. Profitable discussion has been observed, arising from individual computer applications, but all too often an overambitious approach leads merely to copied notes or short paragraphs or writing which are too superficial to be of any value.

82. These courses are not usually intended to last for a full academic year. Most of those which have been seen are designed to last for no more than one term, perhaps with one lesson per week. Another arrangement is to concentrate a number of lessons into a two- or three-week period. Both strategies enable those teachers who have the knowledge to give courses covering a large number of pupils. In some schools teachers from a wide variety of subjects have expressed interest in the work and as a result the planning is involving much interdisciplinary collaboration. This is a most welcome development.

83. Schools differ on the question of how much programming to include in these courses. One independent school certainly intends the induction courses to include enough programming to equip the pupils with a skill which can be applied subsequently in various subject areas, and a number of other schools are planning on similar, although less ambitious, lines. At the other extreme some schools will include no programming at all at this stage. These may overestimate the difficulties of teaching pupils to program. There is nothing to gain if programming is taught as a technique devoid of application, but there is no reason why it should be taught in this way. A comparatively small knowledge of programming can be of great value in mathematics. When familiarisation courses are planned the relationship of

any programming component to the mathematics syllabus and its subsequent use throughout the mathematics course need to be considered very carefully.

Mathematics and other subjects

84. If the curriculum is considered in terms of areas of experience then the area of mathematical experience must include mathematics, statistics and some aspects of computing. A common pattern in further and higher education is to have a faculty containing mathematics, statistics and computing. There is growing use in many countries of the term 'the mathematical sciences', and this phrase may indicate a perspective which schools should consider. Mathematics departments in schools are faced with a bigger task than simply preparing pupils for particular subject examinations. In some ways the planning of mathematics in a school requires a style of thinking more appropriate to a faculty than to a single department.

85. There is long experience of the difficulties of establishing a proper relationship between the teaching of mathematics and science in schools, but unless the teachers of both subjects make continuing efforts to cooperate the pupils lose. More recently the need for liaison has extended to subjects (such as biology and economics) which use statistics and to subjects (such as geography) which use looser structural models. Relationships between mathematics and technical drawing may not always have been easy in the past, but recent changes in mathematics syllabuses have produced further problems of liaison; on the one hand geometrical constructions which were taken for granted in the past are no longer taught, on the other hand innovations such as the teaching of geometrical transformations may be unexploited.

86. It is not for the teacher of mathematics to insist on particular methods when the subject is applied elsewhere, but expertise is required within the department to give advice when it is sought and to answer questions which may arise in the minds of the pupils. It is common experience that pupils lack the ability to apply knowledge unless teachers draw their attention to the links between subjects.

87. There is much popular confusion about the scope of mathematics, and there is confusion between mathematics and arithmetic. Arithmetic is an essential part, but only a part, of mathematics. Claims that something that is being taught 'is not mathematics' are often based on very inadequate definitions of the subject. Mathematics nowadays covers a wider range of knowledge than it did a few decades ago, and it is not realistic to expect every mathematics teacher to be well versed in every aspect of the subject which arises in the school curriculum. It is however highly desirable that the team of mathematics teachers in the school should cover these aspects in full.

88. O-level mathematics syllabuses (for example) do not usually provide a wide enough definition of 'mathematics'. There may well be mathematical ideas which the pupil encounters in other subjects which are inadequately covered by his examination syllabus or which are not covered at all. Unless the mathematics teaching in a school has a proper relationship with the uses of mathematics in other subjects the needs arising from the analysis of the curriculum in terms of areas of experience may not be met.

89. The problems of liaison with existing subjects have been known, although not solved, for a long while. Visits to schools show that similar problems are arising in relation to newer areas of the curriculum. The newer subjects particularly concerned are control technology, the related craft-design-technology and computer studies. When the use of computers is involved the problems are especially complex because the question often concerns more than the reciprocal relationship between mathematics and computing—it concerns the relations of both to a third. Still broader problems arise concerning the relations of mathematics and computing with the arts and the humanities. Nevertheless, if current opportunities are well used the use of computers could bring departments closer together.

Does the use of computers involve mathematics?

90. There are many uses of computers at the present time (for example word processing or file handling) where the user is not himself involved in mathematics and where no benefits are to be gained from analysing what is done in mathematical terms. These situations, incidentally, do not usually

involve computer science either. Computer science is concerned not merely with the use of computers but with the machines and the techniques themselves, and in further and higher education it is a discipline with a sizeable mathematical component. As with physics, engineering and statistics the study cannot be pursued in a serious way without eventually coming to terms with the relevant parts of mathematics.

91. There are currently numerous uses of computers in schools which at first sight do not seem to involve mathematics; but on closer scrutiny a different picture emerges. A simulation in science or geography may not require any mathematics if it is simply used by the pupil for some limited purpose, but if the enquiry is pursued more deeply fresh questions arise. Is the simulation acceptable? Is it sufficiently accurate? What is the mechanism of the simulation? Can we improve this simulation, or design one of our own? A discussion of any of these questions quickly involves the pupils in mathematical questions, whether they recognise it or not.

92. File handling has been quoted as a type of computer use which does not involve mathematics. So far it has proved very difficult to find in schools any cases of large files of information being used in a way in which the computer gave significant advantage. Such uses may develop in the future, but at the moment they are talked about far more than they are practised. The interrogation of historical records (for example) may involve no appreciable mathematics, particularly if the interest centres on individual cases. However, such studies easily move on to questions such as—how many people were involved? Is this number important? Is there any link between . . .? and these questions are statistical. The inference of conclusions from statistics is highly dependent on mathematics although it has other aspects as well.

93. Computers and related devices are in use in control technology, and work can be seen in schools at very different levels of sophistication. BIGTRAK (a toy vehicle controlled by an inbuilt computing device) is used in both primary and secondary schools. During visits to two schools a few days apart essentially the same lesson was seen in a nursery class and with a first year secondary group. This may indicate how much is still to be learnt about fitting these new materials into the curriculum. In both cases the pupils pressed a number of controls in sequence and watched what happened.

The nursery class displayed rather more excitement and took special pleasure in feeling the toy creep over their hands. Neither class displayed noticeable foresight nor any wish to direct the machine in a determinate way, although in the case of the secondary school it was clearly the intention of the course to develop these qualities as time went on. But this is a context from which mathematics grows as well as any other developments which the teacher may have in mind. Some existing mathematics courses provide a language and a set of conceptual patterns designed to handle situations such as these.

94. As one of the longer term issues schools will have to come to terms with the relationship between computer languages and natural language. At the trivial level the English teacher may need to understand why a group of pupils suddenly starts writing GOTO as one word, or why they start talking about "syntax" when he does not use the word himself; but there are much deeper issues. Are computer languages really languages at all, or are they no more than methods of coding? Present day languages are far more than codes. Codes are not designed as aids to thinking, and they are usually of little help in concept formation. Computer languages assist certain styles of thinking and impede others, and a pupil's power to program is part of his general language capability. The introduction of a computer language concerns teachers of more than one subject.

95. Whatever may develop in the future the present choice of language is very limited. BASIC has been much criticised but it is the only practical option at the moment and it has some very good qualities. Very few alternatives have been seen in schools. The implementations of LOGO to be found in schools seem to offer only turtle graphics. A group of teachers visited in a university department of education was deeply involved in the language PROLOG. This group involved teachers from very diverse disciplines, and the study of what was for all of them a new computer language led them to consider one another's problems from a quite fresh point of view, and gave them a sense of common purpose. This could indicate an important new line of development.

96. So we see that many of the computer applications which have been seen in schools are not immediately mathematical, but they quickly involve mathematics if they are followed through. The use of computers is not

necessarily mathematical; the *reflective* use frequently is. If we analyse the curriculum of the pupil in terms of areas of experience we need to consider mathematics not only as a course defined by a syllabus but also as an activity in many areas of the curriculum; and it is part of the professional expertise of the mathematics staff in a school to relate to all of this activity.

97. An article in a journal on computer education recently contained the proposition, "Computing skills are useful far beyond the mathematics class, but only in the way that mathematics itself should be". This proposition may not be universally valid but much can be learnt by discussing it.

Implications for mathematics teaching

98. Microcomputers have contributions to make at almost all levels of the teaching of mathematics, but some developments are much more easily obtainable than others and it is necessary to consider not only possibilities but priorities.

99. The microcomputer has been used in mathematics teaching in a variety of modes, and the degree of conscious interaction which the pupil has with the machine varies greatly. At one extreme the pupil follows the machine's instructions and he is not involved at all in how the machine works or how its program is constructed; at the other extreme the pupil decides what is to happen—he recognises a problem, analyses it and takes a program through progressive stages of refinement until it achieves his purpose. Whilst profitable use can be made of computers throughout this spectrum the reactions of pupils seem strongly related to the extent to which they are mastering the medium as well as learning other ideas. The computer then becomes more than an aid, it is also an object of study. The pupils who have displayed unusual enthusiasm and involvement have been directing the machine more than the machine has been directing them. This is crucial when planning further developments.

100. In primary schools the issues are complex. There the curriculum is less compartmentalised and progress is very dependent on the production

of suitable software and on the extent to which teachers can cope with the new ideas. As suitable programs are written many new uses may develop, but only comparatively few children of primary age have so far shown the ability to forge ahead with confidence and use the computer for their own purposes. The seven year difference at the age of 11 to which the Cockcroft report refers certainly applies to this activity, and whilst a number of children of this age show aptitude and should be encouraged others are only confused if they are urged beyond their capacity. In the near future it is doubtful if even a majority of primary schools will have teachers with sufficient confidence to help children to develop in their own way. There is no value in children simply struggling with techniques at this stage. It is a further problem that if programming is started in the primary school there may be no guarantee of adequate follow-up as the children move on to secondary school. The most promising approach may therefore be to develop programs or packages which enable children to explore ideas for themselves: one way being to offer a rich but restricted language within which they can work and obtain a relatively quick reward.

101. At secondary level the possibilities are immense. As with the adoption of calculators some schools are already making major changes but the majority are waiting to see what develops. Previous investigations have shown that most teachers do not base their teaching aims on their own judgment of what constitutes mathematics, they base them on received opinions derived above all from examination syllabuses and textbooks, and also on public expectation. Many teachers have accepted the view that they cannot do anything until other people have developed appropriate software, but to a mathematician a microcomputer, with an in-built BASIC interpreter, is wealth unimaginable even a short time ago.

102. Various national groups have begun to develop material, some of which is funded by the Microelectronics Education Programme. This material involves, for the most part, large programs providing some particular facility. The big gap in present classroom developments is in the lack of appreciation of the value of small, modest programs which clarify simple pieces of quite ordinary mathematics.

103. Some teachers are producing very creditable, comparatively ambitious programs using the 'electronic blackboard' for class teaching,

and there are many useful programs of the kind produced by resource centres. These have their value, but the potential of the short program is insufficiently appreciated. It is most instructive to see how the greatest common divisor (highest common factor) of two numbers can be produced by repeatedly subtracting the smaller from the larger; how the ratio of successive terms in a Fibonacci series tends to a limiting value; how a comparatively small change in the rate of compound interest makes a surprising difference after a few years. Such programs can make mathematical points very clearly although they may be so short that it is not worth storing them on tape or disc. It should be part of the equipment of every mathematics teacher to be able to construct such programs on the spot—but it is just as important to develop in pupils the power to do so. There are programs of this kind relating to almost any part of traditional algebra and arithmetic, and to many parts of geometry as well.

104. There is long experience that many pupils have great difficulty with the early stages of algebra because they cannot see the point of using letters. (They do not usually voice this difficulty, because they believe they are expected to suffer in silence.) The computer provides an immediate motivation. Variables have to be included in a program to handle whatever data is subsequently fed in; and the reason why you must work this way is clear. The value of this approach to algebra is so apparent in the few places in which it has been tried that it should be considered much more widely, even in courses where algebra will be examined at the end by traditional questions.

105. Where schools are already using microcomputers in their mathematics teaching it is remarkable how much of the work uses the graphics resources of the computer. Other teachers who are wishing to begin might learn from this and give high priority to mastering the early stages of computer graphics.

106. Computers are extremely useful for handling data in science or in the humanities, and in clarifying the theoretical ideas of statistics such as variance and the normal distribution. These legitimate pieces of number-crunching are still very hard to find in schools and this is an area of work which seems ready for early development.

107. Sooner or later the question arises "should mathematicians teach programming?" It is likely that in a few years time the majority of mathematics syllabuses will include the early stages of mathematical programming. This raises several questions. How much should be included, and when? What is the purpose of including it? Is it to clarify the mathematics which is studied already? Is it for application elsewhere? Is it technical training?

108. In schools where computer studies are taught as a separate examination subject there may be differences of view which have to be resolved. It will be said that mathematicians do not write programs properly; little programs to illustrate points of algebra do no more than develop bad habits; introducing programming in a mathematical context gives a false perspective of present day usage. These matters need to be discussed, and discussion can clarify the aims in teaching both subjects.

109. These questions are small when compared with a major phenomenon which is taking place in many schools at the moment—the astonishing response of the pupils to computers if they are allowed the opportunity to use them in their own way and to decide on directions for themselves. Excitement is not new in mathematics lessons, but it has not been common. The inspiration which pupils find in the computer is remarkable in two ways. It is remarkable that pupils will work on tasks for so long and with such intensity. Cases have been known before of rare teachers with the gift of evoking a response beyond all ordinary expectations, and cases of individual pupils displaying uncommon commitment on their own initiative—what is remarkable at the present time is that so many pupils should display commitment in so many schools, and in schools which have no discernible special advantages.

110. The outstanding implication for the teaching of mathematics is that this energy should be developed. The pupils may not have recognised the mathematical aspects of what they are doing; it is for schools to generate direction and continuity from these spontaneous beginnings. Bridges have to be built between the activities which pupils enjoy and the more formal parts of the mathematics course. The pupils who have so far displayed such remarkable enthusiasm may be the ones who can generate problems of their own if they are given the opportunity. A more structured approach might

enthuse a further band of pupils who do not have quite the same originality; but it is essential not to kill the spirit. If the approach is too serious opportunities are lost. If teachers show no sense of fun or if the computer room is open only to 'serious' users or to pupils of examination courses the spirit does not build up. Already there are schools where the pupils' initial enthusiasm is waning. This is usually because the activity has become institutionalised in the wrong way and the pupils are no longer making decisions themselves. In some other schools computers have arrived but the pupils have never enjoyed access. There have been problems of finding a suitable room, or problems of security, and there have been schools where the staff have felt that they needed to understand the machines better themselves before allowing the pupils to get started.

111. In some areas of the country the number of home computers already exceeds the number in schools. If mistaken zeal leads schools into over-organising computer work, with courses in which you are not allowed to do B before you have shown that you can do A, and in which there is so much specified material to cover that you are not allowed to loiter and pick the flowers on the way, pupils will cease to enjoy the computer at school. They will find the excitement elsewhere and education will lose.

Implications for resources, staffing and in-service training

112. The developments now taking place pose many problems when staffing and providing resources in a school, and when planning in-service work for teachers.

113. In a number of schools a teacher has been given the responsibility of developing the use of the school's own microcomputers in a variety of ways. Other schools have appointed a group of senior staff to undertake this task collectively. The experience of many schools has shown the value of having one teacher in charge of the resources, but the latter method seems to offer a better chance of cross-disciplinary work arising. The organisational problem is to combine the advantages of both approaches.

114. A common practice is to locate the school's first batch of machines in a specially assigned room. At the best this means that perhaps fifteen machines are placed in permanent working positions in a room which can be

made secure. Full classes can be taught with two pupils to a machine. Under less favourable circumstances there is a small number of machines and/or a teacher has to transfer the machines from a store to their working positions every time they are used, and return them to the store afterwards. Much of the very successful development work which has taken place so far has depended on teachers undertaking work far beyond the normal calls of duty, and it is not reasonable to suppose that such methods can become widespread practice.

115. It is necessary to ensure that the school's computers are adequately housed and that the need for security does not place unreasonable strain on the staff. If the school is to provide scope for pupils' enthusiasm the computers have to be readily available, without undue fuss and constraint, far beyond ordinary lesson time. In some schools the growth of examination courses in computing threatens to make the computers less available to the enthusiasts who use them outside lesson time, because the machines will be reserved for those who are working on projects. It is to be hoped that sufficient machines will be available to support both types of user. Both activities involve the staff in extra supervisory duties and LEAs and schools need to decide on a policy to make this time available.

116. If the machines are housed in a special room, as many schools are currently planning to do, problems may arise in ensuring that the machines are available as subject teachers require them. Some schools intend to have a few machines which may be transported from room to room on demand; other schools realise that if use develops as they intend one computer laboratory will not be enough for very long. It is likely that particular problems may arise in the teaching of mathematics because the best way to use machines which has been seen so far is to integrate them naturally with general mathematical work. Many years ago it was customary to regard arithmetic, algebra and geometry as separate subjects, perhaps with different slots in the timetable or even different teachers. It took considerable professional effort to bring about the present unity and it would be retrograde to see computing as something which mathematicians can only do on a Friday.

117. In some schools the plan is to allocate a number of individual machines to separate departments. Many mathematics teachers have

shown how useful a single machine can be in a mathematics room, but many more will need a lot of help if they are to see how one machine can fit in with their teaching style and benefit the work done.

118. Some inconvenience is inevitably caused by machines breaking down. Very little information is available on the long term reliability of today's machines. So far experience in the field has shown numerous inconveniences but no major disasters. Some LEAs have been able to provide schools with relatively strong support from centres. These centres have sometimes operated from further education colleges. In other authorities the back-up is much less certain. In all cases schools are very dependent on outside help; in some cases science laboratory technicians or audio-visual technicians have taken on extra work maintaining computers, but generally speaking there is very little technical support to be found within the school if anything goes wrong.

119. The Cockcroft report drew attention to the likelihood that a significant proportion of the teaching of computer studies will be undertaken by mathematics teachers, who will in consequence have less time available to teach mathematics. More recent experience serves only to underline this warning. Some new entrants to teaching have specialised in computer science rather than mathematics, but they are still a small proportion of the whole. In the majority of cases computer studies are being taught by teachers who might otherwise be teaching mathematics and/or science. In schools where rolls are decreasing the problems of staff deployment are further accentuated. Mathematics and science are classified as 'priority subjects' whereas, at the moment, computer studies are not.

120. The Cockcroft report went on to say "it is probably also the case that mathematics teachers who have displayed the initiative to develop computer studies courses are likely to be among the more enterprising and effective teachers . . .". Recent visits to schools confirm this view. Although many computer studies courses are unimaginative and make little appeal to pupils who are hoping for something practical, up to date and exciting they can offer opportunities to work in a new way, and it is quite common to find the computer studies in charge of a younger teacher of mathematics who reacts against the stereotyped mathematics usually found in examination courses and who finds challenge and an invitation to fresh thinking in a

new area of work. Many of the things which such teachers put into their computer studies could very profitably find a place in mathematics courses. There is the additional consideration that computer studies courses are almost always optional courses taken only by a minority of pupils, whereas mathematics is taken in some form by all and consequently the development of interesting courses demands high priority. It is wasteful of limited resources if the depressing image of mathematics in a school drives enterprising teachers of the subject to seek new opportunities in what they see as a separate area of the curriculum. When mathematics syllabuses are designed, and when mathematics teachers are trained, a welcoming attitude to computer work is essential if opportunities are not to be lost.

121. There is a further factor to consider when mathematics teaching by established methods is replaced by computer work. A great many of the classes inspected doing computer work have been unusually small. In some cases this has been because the number of machines was limited and the teacher had been given a half class only while he got the course started. But even in schools where computer courses have been established for some time they are part of the option system and they follow the pattern observed in the National Secondary Survey—the options were taught in smaller groups than the compulsory subjects.

122. As further indication of the tendency to teach computer studies in smaller groups we may note the recommendations of the Scottish Education Department. They recommend that computer studies should be taught in classes of not more than 20 and that a minimum of four machines should be available.

123. We may note also the expressed intention of a school included in the new Technical and Vocational Education Initiative to teach the option subjects on their TVEI courses with small classes of 12 to 15 students. These same students may expect to study mathematics (perhaps with the same teachers) in classes of more than double the size. These cumulative pressures on teachers of mathematics will be particularly hard to bear where they try to increase the practical component of their courses (as recommended in the Cockcroft report) and to introduce more computer work into the day to day teaching of their subject.

124. Above we described the short courses of computer familiarisation which are now being developed in the early years of the secondary school. The experience of schools which are moving in this direction is that a great deal of consultation is required at the planning stage. In-service training within schools has met with varying success, but increasingly schools are recognising the need to undertake this kind of work. This requires a lot of time, and unfortunately cases have been seen of heads of mathematics departments who were so occupied with introducing computers to other members of staff that they had not found time to consider simple and direct uses of microcomputers in their own subject. However, the task of integrating computer work with the rest of the curriculum provides an excellent opportunity to bring together teachers from different subjects with a common purpose.

125. There is no reason to believe that today's microcomputers are the end of the line. All the signs are that technical developments continue at an undiminished pace. Consequently the introduction of microcomputers into schools will produce in-service training needs which will be substantial and which will continue for a long time. Mathematics is one of the subjects most affected and the provision of in-service courses for mathematics teachers demands high priority. Many LEAs and training institutions have made substantial efforts and are continuing to do so, but when the turnover of teachers on courses is compared with the overall numbers involved in the teaching of the subject it is very clear that needs are a long way from being met.

126. When in-service courses on microcomputers are planned the needs of mathematics teachers differ in at least one important aspect from the needs of teachers in other subjects. Whilst it is very important that the national effort to develop good mathematical software should continue, the most important thing of all is that pupils should be helped to make the machine do what they want it to do. Pupils are learning more mathematics from programs which they have written than from programs which others have written for them—and the indications are that this will continue.

127. The natural ability of some pupils is so great that they are mastering the machine far more rapidly than their teachers. Computers demand the development of new skills where you cannot always rely on previous

knowledge and experience. But if larger numbers of pupils are to make similar progress teachers need to know enough to provide the right kind of encouragement. A suitable initial objective (for teachers and pupils) is to develop the confidence to write short and simple programs, or to modify existing programs, as circumstances require. Furthermore, this ability needs to be deployed right across the mathematics curriculum. The in-service training of mathematics teachers in computer work should give this aspect the highest priority. Teachers also need to become familiar with a range of programs offering standard mathematical procedures for statistics, graphical drawing etc., and the teaching packages which adopt new-styles of presentation on an 'electronic blackboard'. After this individual teachers may be ready to extend their own programming ability in BASIC and in assembly code (and perhaps other languages) so that they can cope with the questions which pupils are asking increasingly.

128. Some very exciting work with computers has been seen in schools, but it is not cheap in hardware resources or in staff. If other areas of the curriculum were provided with fresh equipment at equivalent cost, if teaching groups were cut to a similar size, and if the teachers put in the same extensive preparation remarkable results might also be achieved in these areas; but this is not an available choice. Computer work is receiving direct funding and official encouragement which other curriculum areas are not, and there is moreover an indefinable feeling that the future lies this way.

129. In recent decades a number of educational developments have shown great promise when they were undertaken in a limited number of schools in close contact with the sources of the material they were using; but disappointment resulted when wider dissemination was attempted.

130. Some of the currently advocated uses of microcomputers might meet with a similar fate. However, in one important respect the present innovations are not to be compared with superficially similar innovations in the recent past. The new developments are linked with a major, irreversible advance in technology; the problem for education is not only to use them, but to teach about them and to equip some young people to extend the technology in the future. Some areas of the curriculum may be concerned only with one or two of these aspects of computing, but science, technology and mathematics, are concerned with all three. Furthermore, in these areas

(and perhaps in some others) we are seeing young people learning by using computers whether the school curriculum encourages them or not. If full advantage is to be taken of the present educational opportunities the needs in resources, staffing and training must be recognised and the costs met.